HW10

1| Let V be an i.p.s. and $T \in End(V)$. Show that T is normal if one of the following conditions is fulfilled

(1) $T = T^{+}$

(2) $T^{-1} = T^{+}$

(3) $T = -T^{+}$

2| Give an example of an operator that is normal but not diagonalizable.

3| Let V be an ips. $T \in End(V)$ normal. Show

(1) $\|T(x)\| = \|T^{+}(x)\| \quad \forall x \in V$

(2) $T - \lambda id_V$ is normal for $\lambda \in F$.

(3)
$$E_\lambda(T) = E_{\bar{\lambda}}(T^{+}) \quad \text{for all } \lambda \in F.$$

(4) If $\lambda_1 \neq \lambda_2 \in F$, then $\quad E_{\lambda_1}(T) \subseteq \left(E_{\lambda_2}(T)\right)^{\perp}$.

4| Let V be an i.p.s. $T \in End(V)$, s.t. $T = T^{+}$. Show that every eigenvalue of T is real, using 3(3).

5| Let V be an i.p.s., $T, U \in End(V)$, s.t. $T = T^{+}$, $U = U^{+}$. Show that
$$TU = (TU)^{+} \iff TU = UT.$$

6| Let V be an i.p.s. $T \in \mathrm{End}(V)$, $W \subset V$ a T-invariant subspace. Prove:

(1) $T = T^{+} \implies T|_W = (T|_W)^{+}$

(2) $W^{\perp}$ is T^{+}-invariant

(3) If W is also T^{+}-invariant, then $(T|_W)^{+} = (T^{+})|_W$