Practice Midterm 1

UCLA: Math 31A, Fall 2017

Instructor: Jens Eberhardt
Date: 08 October 2017

- This exam has 4 questions, for a total of 16 points.
- Please print your working and answers neatly.
- Write your solutions in the space provided showing working.
- Indicate your final answer clearly.
- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.
- Non programmable and non graphing calculators are allowed.

Name: ______________________________

ID number: ______________________________

Discussion section (please circle):

Day/TA	Allen Boozer	Ben Szczesny	Fan Yang
Tuesday	1A	1C	1E
Thursday	1B	1D	1F

Question	Points	Score
1	4	
2	4	
3	4	
4	4	
Total:	16	

1. Consider the following function

$$f(x) = \begin{cases} x^2 + x + 1 & \text{if } x \leq 3 \\ \sqrt{6x+7} & \text{if } x > 3. \end{cases}$$

(a) (2 points) Using the limit laws, determine the left-hand and right-hand limit of $f(x)$ at $x = 3$.

(b) (1 point) Does the limit of $f(x)$ at $x = 3$ exist?

(c) (1 point) Is $f(x)$ continuous at $x = 3$? If not, which type of discontinuity does it have?

Solution:

(a) By the limit laws we have

$$\begin{aligned} \lim_{x\to 3^-} f(x) &= \lim_{x\to 3^-} (x^2 + x + 1) = \lim_{x\to 3^-} x^2 + \lim_{x\to 3^-} x + \lim_{x\to 3^-} 1 \\ &= (\lim_{x\to 3^-} x)^2 + 3 + 1 = 3^2 + 3 + 1 = 13 \end{aligned}$$

and

$$\begin{aligned} \lim_{x\to 3^+} f(x) &= \lim_{x\to 3^+} \sqrt{6x+7} = \sqrt{\lim_{x\to 3^+} (6x+7)} \\ &= \sqrt{6 \lim_{x\to 3^+} x + \lim_{x\to 3^+} 7} = \sqrt{18+7} = 5 \end{aligned}$$

(b) No, since the left-hand and right-hand limit of $f(x)$ at $x = 3$ are not equal.

(c) The function $f(x)$ is not continuous at $x = 3$ since its limit at $x = 3$ does not exist. It has a *jump discontinuity* at $x = 3$.

2. (4 points) Determine the indeterminate form and compute the following limit algebraically

$$\lim_{x\to 1}\left(\frac{x+5}{x^2+x-2}-\frac{2}{(x-1)}\right)$$

Solution: The limit has indeterminate form $\infty-\infty$ at $x=1$. Now

$$\begin{aligned}\frac{x+5}{x^2+x-2}-\frac{2}{(x-1)} &= \frac{x+5}{(x-1)(x+2)}-\frac{2}{(x-1)}\\ &= \frac{x+5}{(x-1)(x+2)}-\frac{2(x+2)}{(x-1)(x+2)}\\ &= \frac{x+5-2(x+2)}{(x-1)(x+2)}\\ &= \frac{-(x-1)}{(x-1)(x+2)}\\ &= \frac{-1}{x+2}.\end{aligned}$$

Hence, using continuity, we get

$$\lim_{x\to 1}\left(\frac{x+5}{x^2+x-2}-\frac{2}{(x-1)}\right)=\lim_{x\to 1}\frac{-1}{x+2}=\frac{-1}{1+2}=-\frac{1}{3}.$$

3. Consider the function

$$f(x) = x^3 + 1$$

   (a) (3 points) Compute $f'(1)$ using the definition of the derivative. You are *not* allowed to use the power rule!

   (b) (1 point) Determine the equation of the tangent line of $f(x)$ at $x = 1$.

Solution:

(a) By definition

$$f'(1) = \lim_{h \to 0} \frac{f(1+h) - f(1)}{h}.$$

We compute

$$\begin{aligned} \frac{f(1+h) - f(1)}{h} &= \frac{(1+h)^3 + 1 - (1^3 + 1)}{h} \\ &= \frac{1 + 3h + 3h^2 + h^3 + 1 - 2}{h} \\ &= \frac{3h + 3h^2 + h^3}{h} = 3 + 3h + h^2. \end{aligned}$$

Hence

$$f'(1) = \lim_{h \to 0} (3 + 3h + h^2) = 3.$$

(b) The equation of the tangent line is given by

$$y - f(1) = f'(1)(x - 1)$$

which is

$$y - 2 = 3(x - 1).$$

4. Compute the following derivatives. You may use all rules learned so far.
   (a) (2 points) $\frac{d^2}{dx^2}(3x^3 + 4x^2 + 2x - 1)$
   (b) (2 points) $\frac{d}{dx}\frac{x^2+1}{2x}$

Solution:

(a) We have

$$\frac{d}{dx}(3x^3 + 4x^2 + 2x - 1) = 9x^2 + 8x + 2 \text{ and}$$
$$\frac{d}{dx}(9x^2 + 8x + 2) = 18x + 8.$$

Hence

$$\frac{d^2}{dx^2}(3x^3 + 4x^2 + 2x - 1) = 18x + 8.$$

(b) By the quotient rule

$$\frac{d}{dx}\frac{x^2+1}{2x} = \frac{2x2x - (x^2+1)2}{(2x)^2} = \frac{2x^2-2}{4x^2} = \frac{x^2-1}{2x^2}.$$

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