HW 7,8

1| Consider $T = L_A : \mathbb{R}^3 \to \mathbb{R}^3$, where

$$A = \begin{pmatrix} 1 & 1 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 3 \end{pmatrix}.$$

(a) Determine $\chi_T(t)$ and the eigenvalues of T.
(b) Determine the eigenspaces of T.
(c) Determine the algebraic and geometric multiplicities
(d) Is T diagonalizable? If yes, determine a basis of eigenvectors.

2| Let $T: V \to V$ be a linear operator. Prove or disprove
(1) T diagonalizable $\Rightarrow$ T^2 diagonalizable
(2) T^2 diagonalizable $\Rightarrow$ T diagonalizable.

3| Let $T: V \to V$ be a diagonalizable operator on a finite dimensional vector space. Determine $\det(T)$ and $tr(T)$ in terms of the eigenvalues of T and their multiplicity.

4| Let $T: V \to V$ be an <u>invertible</u> linear operator. Show that $\lambda \in F$ is an eigenvalue of $T \Leftrightarrow \lambda^{-1}$ is an eigenvalue of T^{-1}!

5| Let β be a basis of a finite-dimensional inner product space V. Show that if $\langle x, z \rangle = \langle y, z \rangle$ for all $z \in \beta$, then $x = y$.

6| Let $\langle\, , \rangle_1$ and $\langle\, , \rangle_2$ be two inner products on a vector space V. Show that $\langle\, , \rangle_1 + \langle\, , \rangle_2 : V \times V \to F$, $(x,y) \mapsto \langle x, y \rangle_1 + \langle x, y \rangle_2$ is also an inner product on V.