1] Let $X = \{1,2,3\}$, $F = \mathbb{Q}$ and $V = \mathrm{Fun}(X,F)$.
Let $\beta = \{\mathbb{1}_1, \mathbb{1}_2, \mathbb{1}_3\}$, where

$$\mathbb{1}_x(y) = \begin{cases} 1 & x = y \\ 0 & \text{else} \end{cases}$$

and $\gamma = \{\overline{\mathbb{1}}_1, \overline{\mathbb{1}}_2, \overline{\mathbb{1}}_3\}$, where

$$\overline{\mathbb{1}}_x(y) = \begin{cases} 0 & x = y \\ 1 & \text{else} \end{cases}$$

Compute $[id_V]_\beta^\gamma$ and $[id_V]_\gamma^\beta$

2] Let $V = \mathbb{R}^2$, $\alpha, \beta \in [0, 2\pi]$ be angles. Proof that

$$R_\alpha R_\beta = R_{\alpha+\beta}$$, where for θ

$$R_\theta : \mathbb{R}^2 \to \mathbb{R}^2, \quad \begin{pmatrix} x \\ y \end{pmatrix} \longmapsto \begin{pmatrix} x\cos(\theta) - y\sin(\theta) \\ x\sin(\theta) + y\cos(\theta) \end{pmatrix}$$

is rotation by θ degrees.

3] Let F be a field $A \in F^{n\times m}$, $B \in F^{m \times n}$. Show that

$$tr(AB) = tr(BA)$$

4] Let V be a f.d.v.s. over a field F, β, γ bases of V. Let $T: V \to V$ be linear. Show that

$$tr([T]_\beta) = tr([T]_\gamma).$$

4| Let V be a f.d. v.s. over a field F. $T: V \to V$ a lin. op..
Show that for $\lambda \in F$

$$\lambda \text{ is an eigenvalue of } T \iff \chi_T(\lambda) = 0.$$

5| Let V be a v.s. over a field F, $W_1, \dots, W_m \subset V$ subspaces.
Then V is called the direct sum of $W_1, \dots, W_m$, written

$$V = W_1 \oplus \cdots \oplus W_m \quad , \text{if}$$

(1) $W_i \cap \left(\sum_{\substack{j=1 \\ i \neq j}}^{m} W_j \right) = \{0\}$ for all $1 \leq i \leq m$, and

(2) $V = \sum_{j=1}^{m} W_j$.

Assume that V is finite dimensional. Assume that $W_1, \dots, W_m$ fulfill (1). Show that

$$\dim(V) = \sum_{j=1}^{m} \dim(W_j) \iff V = \sum_{j=1}^{m} W_j .$$