Practice Midterm 1
UCLA: Math 115A, Fall 2017

Instructor: Jens Eberhardt
Date: 08 October 2017

- This exam has 4 questions, for a total of 16 points.

- Please print your working and answers neatly.

- Write your solutions in the space provided showing working.

- Indicate your final answer clearly.

- You may write on the reverse of a page or on the blank pages found at the back of the booklet however these will not be graded unless very clearly indicated.

- Non programmable and non graphing calculators are allowed.

Name: _____

ID number: _____

Question	Points	Score
1	4	
2	4	
3	4	
4	4	
Total:	16	

1. Prove or disprove that the following subsets W of the $\mathbb{R}$-vector space $V = \mathbb{R}^3$ are subspaces.

   (a) (2 points)
$$W = \left\{ (a, b, c) \in \mathbb{R}^3 \,\middle|\, a^2 + b^2 + c^2 = 0 \right\}$$

   (b) (2 points)
$$W = \left\{ (a, b, c) \in \mathbb{R}^3 \,\middle|\, a + b + c = 0 \right\}$$

2. (4 points) Let $S = \{(1, -1, 0), (0, 1, -1), (1, 1, 1)\} \subseteq \mathbb{R}^3$. Prove or disprove that S is a basis of $\mathbb{R}^3$

3. Let W_1, W_2 be subspaces of a vector space V over a field F. Prove or disprove that the following subsets are also subspaces of V.

   (a) (2 points) The intersection of W_1 and W_2

   $$W_1 \cap W_2 = \{v \in V \mid v \in W_1 \text{ and } v \in W_2\}.$$

   .

   (b) (2 points) The difference of W_1 and W_2

   $$W_1 \backslash W_2 = \{v \in V \mid v \in W_1 \text{ and } v \text{ is not an element of } W_2\}.$$

   .

4. Let V be a vector space over a field F and let $x, y, z \in V$. Prove the each of following statements or disprove them providing a counterexample.

   (a) (2 points) Assume that

   $$z \in \text{Span}(x, y) \text{ and}$$
   $$x \in \text{Span}(y, z).$$

   Then also $y \in \text{Span}(x, z)$.

   (b) (2 points) Assume that

   $$x \neq 0 \text{ and}$$
   $$x \in \text{Span}(y).$$

   Then also $y \in \text{Span}(x)$.

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